Five experiments addressed the issue of how pigeons learn to discriminate the relative frequency of stimuli. During a sampling period, three different stimuli (keylights) were presented serially, in mixed order, and with different frequencies. During a choice period, the stimuli were presented simultaneously, and reinforcement was arranged for choosing the stimulus that was presented the least number of times during the sample. The results showed that (a) the overall proportion of correct choices was always above chance levels; (b) the likelihood of a correct choice decreased with the serial position of the correct stimulus, a negative recency effect; (c) when the last three stimuli of the sample were constrained to be one of each kind, the negative recency effect decreased but errors became more likely when the correct stimulus occurred early in the sample, a negative primacy effect; (d) accurate performance generalized to new and larger samples; and (e) under some conditions the probability of a correct choice was independent of the serial position of the correct stimulus. The serial position curves suggest that in a least frequent discrimination task, two processes determine how the least frequent stimulus controls behavior: a passive decay process (the stimulus loses its effectiveness with time since its last occurrence), and a residual salience process (when the stimulus occurs in the first position it may decay to a higher asymptote than when it occurs in later positions).

Key words: frequency discrimination, negative recency and negative primacy effects, temporal integration, generalization, key peck, pigeon

The reader would probably have no difficulty in answering the following question: “In the last month, did you eat more often at Chinese or Italian restaurants?” As several studies show, humans can accurately estimate the frequency of occurrence of many events (Crowder, 1976; Greene, 1992). According to some researchers, the results of these studies suggest that frequency is a fundamental, irreducible dimension of stimulus control because knowledge of frequency is automatic (i.e., it comes without the need for feedback, effort, practice, or special instructions), shows no major developmental trends, and varies little across individuals (Hasher & Zacks, 1984).

The large volume of human research on the topic of frequency discrimination contrasts sharply with the paucity of similar studies with animals. This discrepancy is unfortunate, because the study of frequency discrimination may help us understand important issues of learning. First, and most obviously, such studies increase our understanding of the range of dimensions that may control the behavior of animals. Although some studies have addressed the related processes of number and rate discrimination (Davis & Perusse, 1988; Gallistel, 1990; Meckner & Guevrekian, 1962; Meck & Church, 1983; Rilling & McDermid, 1965; Roberts, 1995), the focus of the present paper is the discrimination of a property—relative frequency—that is less precise than number and more specific than rate. A relative frequency discrimination differs from a number discrimination in that it is based on the relative, not the absolute, cardinalities of two or more sets of events. (A number-related question would ask, “In the last month, how many times have you visited Chinese and Italian restaurants?”). Furthermore, when two or more stimuli occur serially and in mixed order, as in a typical frequency discrimination task, the behavioral effect of each stimulus is likely to differ from its effect when presented alone, as in a typical number discrimination task. A relative frequency discrimination differs from a rate discrimination in that it is based on a specific time frame (i.e., “In the last month did
you more often visit Chinese or Italian restaurants?

Second, frequency discrimination may shed light on how animals integrate events over time. This happens because in a frequency discrimination task, two or more stimuli are presented serially, and, therefore, some stimuli are necessarily closer than others to the moment when the animal makes what might be called its frequency judgment. If animals weigh more heavily the last events of a series, for example, their frequency judgments will be biased by the recency of the events. The human counterpart of this frequency-recency confusion would be to overestimate the frequency of visits to Chinese restaurants during the last month because yesterday you ate in a Chinese restaurant (for discussion of the interactions between recency and frequency in human discriminations of recency and frequency see, e.g., Crowder, 1976; Flexser & Bower, 1974; Galbraith, 1976). In addition, when different stimuli are presented serially, time-dependent interactions among them, such as stimulus interference or competition, are likely to take place. Hence, by analyzing the properties of frequency discrimination we may be able to understand better how past events control current behavior.

Third, frequency discrimination may be the main process that underlies more complex behavior. For example, pigeons can learn to discriminate slides that contain human beings from slides that contain no humans (Herrnstein & Loveland, 1964). Apparently, no single feature is either necessary or sufficient to determine the category membership of a slide. It is conceivable, however, that pigeons learn to base their choices on the difference between the two categories in terms of the relative frequencies of clusters of features. Frequency discrimination could therefore be an important process underlying concept learning (Hasher & Zacks, 1984).

Alsop and Honig (1991) performed the only study that directly investigated the discrimination of the relative frequency of events by animals. During a sample period, pigeons pecked a center key illuminated multiple times with either a blue or a red light. During a subsequent period the birds chose between a left and a right key. When the blue stimuli had outnumbered the red, a peck at the left key was reinforced; when the sample had contained more red than blue stimuli, a peck at the right key was reinforced. The results showed that pigeons learned to discriminate the relative frequencies of the two colors. The authors also reported a negative recency effect—the birds tended to overestimate the frequency of a stimulus when that stimulus was the last to occur before the choice period.

However, several issues remain unanswered: Do pigeons choose on the basis of the more frequent stimulus, the less frequent, or both? How does frequency discrimination extend to more than two events or to larger sets of events? Can the birds learn to weigh the stimuli regardless of their order of presentation, or will they always show a negative recency effect? What does the pattern of errors during generalization tests tell us about the properties of frequency discrimination? The experiments described here address these issues.

Our general approach consisted of varying the relative frequencies of three stimuli during a sampling period and then, during a choice period, reinforcing the selection of the least frequent stimulus. The stimuli occurred one at a time during the sampling period, but were presented simultaneously during the choice period. If we designate the three stimuli by the letters A, B, and C, a typical trial with C as the correct choice would have the following structure:

\[
\begin{align*}
&\text{ABBACABBA} \\
&\text{Sampling period} \\
&\text{Choice period}
\end{align*}
\]

By using three stimuli, one with a low frequency and the other two with higher frequencies, we increased the likelihood that the subjects were effectively choosing the least frequent stimulus and not avoiding the most frequent one. By arranging specific orderings of the stimuli during the sampling period, we attempted to understand how different-aged events affect the discrimination. By manipulating the total number of stimuli in the sample, we tried to find out how the discrimination of relative frequency extends to larger sets of events. And finally, by interspersing new samples during a regular session, we
studied how performance generalizes to un-
trained sets of stimulus frequencies.

EXPERIMENT 1

In Experiment 1 we asked whether pigeons
can discriminate the relative frequencies of
three stimuli that differ along two dimen-
sions, spatial location and color. In addition,
during the first phase of the experiment, one
of the stimuli did not occur during the sam-
pling period. We reasoned that the redu-
dancy of the two stimulus dimensions and the
zero frequency of one stimulus would make
the discrimination easier to learn, and thus
would provide a convenient starting point for
our investigations.

When one stimulus does not occur during
the sample, accurate choice performance
may be achieved by discriminating its relative
frequency (zero being one end of the fre-
quency dimension), its relative recency (at
the moment of choice the zero frequency
stimulus is also the least recent, most novel
one), or a mixture of frequency and recency.
To determine to what extent frequency and
recency controlled the birds' performance, in
the next phases of the experiment we pre-
sented the least frequent stimulus once dur-
ings the sampling period. If the discrimina-
tion is based on the relative recency of the stimuli,
then the probability of a correct response
should decrease as the correct stimulus oc-
curs later in the sample—a negative recency
effect. If the discrimination is frequency
based, however, then no such correlation
should occur.

METHOD

Subjects

Three experimentally naive pigeons (Co-
lumna livia), maintained at 80% of their free-
feeding body weights, participated in the ex-
periment. The birds were housed in
individual home cages with grit and water
continuously available. A 12:12 hr light/dark
cycle was in effect in their home cages.

Apparatus

The experimental chamber was 37 cm
wide, 40 cm deep, and 37 cm high. All side
walls and the ceiling were made of Plexiglas,
and the floor was wire mesh. The front wall,
made of aluminum, contained three response
keys, each 2.2 cm in diameter. The keys were
located at the vertices of an equilateral tri-
gle, 5 cm along the sides, with its center 25
cm above the floor and at equal distances
from the side walls. The keys could be illu-
minted from behind with red, green, or
white lights. In this experiment, the left key
was always illuminated with red (R), the right
key with green (G), and the top key with white (W) light. Directly below the keys, a
hopper opening (4.4 by 6.4 cm), 7.2 cm
above the floor, permitted access to mixed
grain. A white light illuminated the hopper
when food was delivered. The houselight, lo-
cated on the top of the left wall, provided
general illumination. The operant chamber
was enclosed in an outer box, on the back
wall of which a ventilating fan provided air
circulation and helped to mask extraneous
noises. An IBM PC® computer controlled the
experiment and recorded the data.

Procedure

Preliminary training. All pigeons learned to
peck the keys by autoshaping training (Brown
& Jenkins, 1968). Sessions terminated after
50 trials, and each trial began with the illu-
mintation of a randomly selected key. If no
peck occurred within 6 s of the onset of the
signal, the keylight was turned off and the
food hopper was raised for 3 s. A peck to the
lit key produced food immediately. A variable
intertrial interval (ITI) averaging 60 s fol-
lowed food presentations. At the end of five
sessions all pigeons pecked the keys reliably.

The birds were then exposed to a transi-
tion phase to train them to respond under
low probabilities of food. As before, each trial
began with the illumination of a randomly se-
lected key. A peck at the illuminated key
turned the keylight off and with probability p
produced 3 s of access to food; with proba-
bility 1 – p the peck produced a 0.4-s delay.
Pecks during the delay reset the timer for the
interval. After the delay or the food presen-
tation, a new trial began. The houselight was
always illuminated except during reinforce-
ment. Across four sessions the probability of
reinforcement per peck decreased from .5 to
.08. Sessions ended after 60 reinforcers. Fol-
lowing the transition phase, the frequency
discrimination training began.

Frequency discrimination training. Each trial
was divided into a sampling period and a choice period. The sampling period began with the illumination of the houselight and one of the three keys. A peck at the illuminated key initiated a 0.4-s delay, during which all keys were dark but the houselight remained illuminated; pecks at a darkened key reset the timer but had no other scheduled consequences. After 0.4 s without a peck, the same or another key was illuminated and the procedure repeated for a total of $N$ stimulus presentations. Following the last 0.4-s delay, the choice period began. All keys were illuminated simultaneously and reinforcement was arranged for choosing the key that was presented (and pecked) the least number of times during the sampling period. A reinforcer consisted of 3-s access to food, during which the houselight and the keylights were turned off. After reinforcement for a correct choice or immediately after an incorrect choice, a 20-s ITI with all lights off followed. A trial with $N = 8$ stimulus presentations and partial frequencies of one, three, and four is illustrated below:

```
20-s blackout → RWWRGWRW → W
intertrial interval sampling period R G
choice period
```

The stimulus frequencies during the sampling period varied across the three phases of the experiment. Table 1 shows the details. During Phase 1, one key was never illuminated during the sampling period (S1 in Table 1), whereas the other two keys (S2 and S3) were illuminated three times each, in random order, for a total of six presentations. Because the size of the stimulus sample was constant, once the frequencies of S1 and S2 were specified, the frequency of S3 was automatically set. During Phase 2, a transition phase, one stimulus was presented zero or one time (the two cases were equally likely), another stimulus three or four times, and the third stimulus was presented the number of times required to make a total of eight presentations. In Phase 3, the least frequent stimulus was always presented once and the other two stimuli were presented three or four times each, for a total of eight presentations. Table 1 shows the number of sessions for each bird and condition; practical difficulties prevented additional sessions for Bird 7380 during Phase 3.

Two additional features of the procedure were designed to reduce any position or color bias. First, after an incorrect choice the trial was repeated with the same sequence of stimulus presentations (correction trials method). Second, the three keys were assigned the same number of times to the frequency classes displayed in Table 1. Specifically, each key was the S1 key on 20 randomly selected trials, the S2 key on another 20 randomly selected trials, and the S3 key on the remaining 20 trials of the session. Hence, sessions ended when the subject collected 60 reinforcers, 20 from each key.

All data analyses exclude the correction trials. The analyses of variance based on correct and incorrect responses were preceded by an arcsine transformation of choice proportions to equalize their variances (Snedecor & Cochran, 1980; ANOVAs with the original proportions always yielded the same results).

**RESULTS AND DISCUSSION**

Figure 1 shows the proportion of correct choices across sessions. All birds learned the discrimination task and, by the end of each phase, the proportion of correct choices was significantly above the one third chance level. In fact, the results from the first session of the experiment were already above the 99% confidence interval associated with random responding. The average proportion of correct choices during the last five sessions decreased across the three phases of the experiment for Birds 2087 (.94, .86, and .82) and 7380 (.90, .85, and .75), but changed little for Bird 2195 (.85, .81, and .82).

When the correct stimulus was not included in the sample, most errors consisted of avoiding the last stimulus. For example, after the sample RRGGRG, most errors consisted
FREQUENCY DISCRIMINATION

Fig. 1. Overall proportion of correct choices during the three phases of the Experiment 1. In Phase 1 the least frequent stimulus (S1) did not occur during the sample; in Phase 2 the frequency of S1 was either zero or one; in Phase 3 the frequency of S1 was one. Under random choice, the proportion of correct trials would be one in three. The vertical lines separate the three phases of the experiment.

of choosing the red key. These errors accounted for 86% (Bird 2087), 83% (Bird 2195), and 67% (Bird 7380) of the total number of errors produced during the last five sessions of Phase 1. This pattern of errors suggests that frequency discrimination was influenced by the relative recency of the stimuli—the stimulus that was not presented in the last position (R in the example above) was less recent than the last stimulus (G) and therefore easier to confuse with the least recent, least frequent stimulus (W).

To see whether recency also influenced the frequency discrimination when the least frequent stimulus occurred once during the sample, we computed the probability of a correct choice for each ordinal position of that stimulus during Phase 3 (the results from the transitional Phase 2 were similar and are not reported). Figure 2 shows that the probability of a correct response decreased with the proximity of the least frequent stimulus to the choice period—a negative recency effect (Alsop & Honig, 1991; Shimp, 1976; Wright, Santiago, Sands, Kendrick, & Cook, 1985). A repeated one-way ANOVA of the data from Phase 3 yielded a strong effect of the serial position of the correct stimulus, $F(7, 14) = 12.5, p = .0001$.

In addition to the serial position curves, the pattern of errors made during Phase 3 also reveals the strong influence of the last element of the sample. Thus, when the correct stimulus occurred in Positions 6 or 7 (e.g., RGGRRWRG or GRRRGGWR), the errors in which the bird avoided the last stimulus accounted for an average of 71% of the total errors (72, 61, and 81% for Birds 2087, 2195, and 7380, respectively). Random performance would predict a value of 50%.

If the shape of the serial position curves and the pattern of errors described above reveal the strong influence of recency, the absolute values of the serial position curves also reveal that recency could not have been the only variable that controlled performance. In fact, exclusive control by recency would predict a proportion of correct choices below, or at best around, the one third chance value whenever the correct stimulus occurred in Positions 7 or 8. As Figure 2 shows, however, even on those trials performance was significantly above chance. We conclude that, as in Alsop and Honig’s (1991) study, both fre-
What explains the negative recency effect observed in Figure 2? One hypothesis is that with each presentation of a stimulus the probability of subsequently choosing that stimulus decreased, but this effect decayed with time. When the correct stimulus was the first of the series, its effect on choice probabilities had more time to decay than when it was the last of the series. Hence, the likelihood of a correct choice decreased with serial position (the negative recency effect).

The preceding hypothesis predicts that without changing the structure of the task (e.g., the nature of the stimuli, the interstimulus interval, and the interval between the sampling and the choice periods), it should be impossible to change the time course of decay of stimulus control, and therefore it should be difficult to eliminate the negative recency effect. In other words, pigeons would always overestimate the frequency of the most recent stimuli. However, it is also conceivable that the negative recency effect depends on the contingencies of reinforcement that are operating in the situation. For example, accurate performance in Phase 1 required memory of at most the last four stimuli. In fact, by listing all possible samples it can be shown that 60% of them required memory of only the last two stimuli—avoiding these during the choice phase would always yield a correct response. Even when the partial frequen-
cies were one, three, and four, as in Phase 3, memory of only the last three or four stimuli would also yield good performance. Hence, the negative recency effect could have occurred simply because the experimental contingencies allowed the last few stimuli of the sample to control the choice response.

If the foregoing contingency-based interpretation is correct, we should be able to reduce the negative recency effect by varying these contingencies in such a way that pigeons weigh more equally events that have occurred at different times. The recency effect, in other words, could be attenuated without changing the structural features of the task. The next experiment tested this hypothesis.

**EXPERIMENT 2**

In Experiment 2 we studied a set of contingencies that were designed to make a frequency discrimination based exclusively on the last three stimuli impossible and, consequently, to increase the control of the choice response by the stimuli presented earlier in the sample. If the contingencies successfully change the birds' behavior, then the probability of a correct choice should not depend on the serial position of a stimulus. In other words, the obtained serial position curves should approach a horizontal line. From a conceptual viewpoint it is also preferable to talk about pure frequency discrimination when the serial position curve is both high and flat; high because a discrimination implies performance well above chance levels, and flat because pure frequency discrimination requires that similar weights be given to events that have occurred at different times.

To discourage a recency-based discrimination, we presented a series of nine stimuli during the sampling period, with partial frequencies of one, four, and four, but with the constraint that the last three stimuli would be one of each type. To perform the task, the bird could not rely exclusively on the last three stimuli because they provided no information about the correct choice. We predicted that this constraint would reduce the negative recency effect and, consequently, that pigeons would weigh the stimuli more equally. To test the prediction, we then eliminated the constraint (i.e., now the least frequent stimulus could appear in any position) and analyzed the serial position curve for errors; without the constraint, the negative recency effect should reappear.

In addition to high and flat serial position curves, frequency discrimination also implies that performance should generalize to new, untrained sets of stimulus frequencies. After training with a set with partial frequencies of one, four, and four, a bird should perform well with sets having frequencies of zero, three, and nine, or two, five, and seven, for example. Without generalization to new sets, we cannot rule out the hypothesis that pigeons learned only specific pattern–response pairings. Hence, the last phase of Experiment 2 included generalization tests.

**Method**

**Subjects**

Five experimentally naive pigeons (*Columba livia*) participated in the experiment. Housing conditions were similar to those described in Experiment 1, except that the pigeon colony was always illuminated.

**Apparatus**

A standard Med Associates operant chamber for pigeons (24 by 29 by 30 cm) was modified to include three keys located at the vertices of an inverted isosceles triangle, 10 cm along the top side and 8.5 cm along the lateral sides. The center of the triangle was located 23 cm above the floor grid. Each key, 2.5 cm in diameter, could be illuminated from behind with red (R), green (G), or blue (B) lights. In this experiment, the left key was always illuminated with red, the right key with green, and the bottom key with blue light. An opening (6 cm by 7 cm), 4 cm above the floor, permitted access to grain. A Sonalert®, attached to the back wall, permitted the presentation of 4500-Hz tones. The operant chamber was enclosed in an outer box. A ventilating fan attached to the box provided air circulation and helped to mask extraneous noises.

**Procedure**

The birds learned to peck the keys according to the preliminary training described in Experiment 1. During the frequency discrimination training, the trial structure remained the same as before, except that the choice
Experiment 2

<table>
<thead>
<tr>
<th>Frequency of S2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency of S1</td>
<td>0</td>
<td>9</td>
<td>8</td>
</tr>
<tr>
<td>Frequency of S2</td>
<td>1</td>
<td>8</td>
<td>7</td>
</tr>
<tr>
<td>Frequency of S3</td>
<td>2</td>
<td>7</td>
<td>6</td>
</tr>
</tbody>
</table>

Table 2

Frequency sets on generalization trials. The cells show the frequency of S3.

period followed the sampling period after a delay of 0.8 s (instead of 0.4 s). In addition, during this delay a 4500-Hz tone was presented. These two changes were intended to increase the distinctiveness of the choice period.

The experiment was divided into three phases. In Phase 1, the sampling period contained nine stimuli with partial frequencies of one, four, and four. The last three stimuli, however, were constrained to be one of each kind, in random order. An example of a sample in which G is the correct response is RRBRBBBGR. No signal separated the first six stimuli from the last three. This phase lasted for 30 sessions, and each session ended when the subject had earned 42 reinforcers, 14 from each key. In Phase 2, the constraint was eliminated and the least frequent stimulus could appear randomly in any position. Sessions ended after 42 reinforcers, and the phase lasted for 15 sessions.

Phase 3 was similar to Phase 2 except that 12 generalization trials were interspersed among the regular trials; on average, one generalization trial followed three regular trials. Table 2 shows the set of frequencies used during the generalization trials. The least frequent stimulus (S1) was presented zero, one, or two times, the intermediate stimulus (S2) was presented three, four, or five times, and the third stimulus (S3) was presented the number of times required to make the sample 12 presentations long. On every generalization session, each key was the S1, S2, or S3 stimulus four times. Incorrect choices on generalization trials did not repeat the trial (the correction procedure was not in effect), but correct choices were reinforced. Sessions ended when the pigeon obtained the 42 reinforcers from the regular trials. Phase 3 lasted for six sessions.

**RESULTS AND DISCUSSION**

By the end of Phase 1 all birds had learned the discrimination task. Their acquisition curves were similar to those obtained in Experiment 1 (see Figure 1) and are not shown. When averaged over the last five sessions, the proportion of correct trials ranged from .68 to .79 in Phase 1, .70 to .84 in Phase 2, and .74 to .85 in Phase 3 (excluding the generalization trials). A one-way repeated measures ANOVA confirmed the slight improvement across phases, $F(2, 8) = 6.86, p = .019$. In what follows we describe the serial position curves, the pattern of errors, and the generalization data.

**Serial position curves.** Figure 3 shows the serial position curves when the constraint was in effect, when the constraint was eliminated, and when sessions also included generalization trials. The curves are averages across birds but also represent individual performance (Appendix A lists the data for each bird). As predicted, with the constraint on the last three stimuli, there was no consistent relation between the likelihood of a correct choice and the serial position of the least frequent stimulus. A one-way repeated measures ANOVA yielded a nonsignificant effect of stimulus position, $F(2, 8) = 1.86, p = .22$. During the first sessions without the constraint, all pigeons showed an inverted U-shaped curve, with both a negative recency effect—a decreased likelihood of a correct choice as the least frequent stimulus occurs earlier in the sample. During the last sessions, the negative recency effect disappeared for all birds except 10490, but the negative primacy effect—a decreased likelihood of a correct choice as the least frequent stimulus approaches the choice period—and a negative primacy effect (Santiago & Wright, 1984; Wright, 1994; Wright et al., 1985)—a decreased likelihood of a correct choice as the least frequent stimulus occurs earlier in the sample. During the last sessions, the negative primacy effect disappeared for all birds except 10490, but the negative recency effect did not change appreciably. A two-way repeated measures ANOVA revealed a main effect of serial position, $F(8, 32) = 5.28, p = .0005$, but no main effect of training, that is first versus last sessions, $F(1, 4) = .15$. The interaction between training and serial posi-
Fig. 3. Average proportion of correct choices during the three phases of Experiment 2 as a function of the serial position of the correct stimulus. In Phase 1 the last three stimuli were constrained to be one of each type. The data are based on the last seven sessions. In Phase 2 the constraint was removed. The filled and open circles correspond to the first eight and last seven sessions, respectively. In Phase 3, generalization tests were interspersed among the regular trials. The data come from all six sessions.

Error patterns. The serial position curves inform us about the control exerted by the correct stimulus at different locations in the sample, but remain silent on the control exerted by the other stimuli. One way to reveal this additional source of control is by analyzing the birds’ error patterns. We call two of these patterns recency-type and primacy-type errors because they inform us about the influence of the other stimuli when they occur in the last and first positions, respectively. A recency-type error takes place when, following samples in which the correct stimulus occurs in Positions 7 or 8, the bird avoids the ninth and last stimulus of the sample. For example, after the samples GGRGRBRG or RGRGRGR...
BG, the bird avoids not only B, the correct choice, but also G, the last stimulus. A primacy-type error takes place when, following samples in which the correct stimulus is the last to occur, the bird avoids the first stimulus of the sample (e.g., after GGRGRRRGB the bird avoids not only B but also G).

Figure 4 shows the birds’ error patterns during the same sessions as shown in Figure 3 (Appendix B provides the raw data). Random performance would predict a proportion of .5 for both types of errors. When the constraint was in effect, the proportion of recency-type errors was close to chance. However, when the constraint was eliminated, these errors increased and then remained high during the last phase of the experiment. A one-way repeated measures ANOVA showed a strong effect of phase of training, $F(3, 12) = 7.8$, $p = .006$. In contrast, primacy-type errors did not depart consistently from the chance level, $F(2, 8) = 0.08$.

With two exceptions discussed below, the pattern of errors and the serial position curves suggest the same interpretation. Thus, both types of analyses indicate that with the constraint in effect during Phase 1, choice behavior was not influenced by the serial position of the stimuli, whereas without the constraint during Phase 2, the last elements of the sample gained differential control over the choice response.

However, the serial position curve revealed a negative primacy effect during the first half of Phase 2, which suggests a relatively strong effect of the first stimulus of the sample, whereas the pattern of errors revealed no such effect. In other words, if the bird tended to avoid the first stimulus as the serial curve for the least frequent stimulus suggests, then we would predict that when the correct stimulus occurred in Position 9 and the bird made a mistake, it would also tend to avoid the stimulus in Position 1. Primacy-type errors should have been above chance, but they were not.

Phase 3 yielded the converse case; the serial position curve showed no evidence of differential control by the last element of the sample, but the pattern of errors suggested otherwise. To see the difficulty in reconciling the two sets of results, consider the following account of recency-type errors: When the bird mistakenly avoided the correct stimulus, it was left with a choice between the two remaining stimuli. Because in most trials without constraint recency was negatively correlated with the correct response, the bird chose the least recent of the two; hence the recency-type errors. But the same logic predicts that errors should have increased when the correct stimulus occurred in Position 9 because its recency would then be maximal; however, they did not.

Although we cannot offer at present a complete solution to the preceding discrepancies, we note that they seem to imply that the effects of serial position may vary with the stimulus. For example, if a stimulus occurs only once in the sample, its influence may be greatest when in Serial Position 1 (say), but the same may not happen to a stimulus that occurs more than once. We will return to this issue in the General Discussion.

Generalization data. Figure 5 shows the results of the generalization tests. The overall proportion of correct choices was clearly above chance levels for all pigeons and showed no consistent trend across sessions. Averaged over sessions and birds, percentage correct was 77.8%.

The top right panel plots proportion correct, averaged across sessions and birds, as a function of the frequency of the least frequent stimulus (S1) and the frequency of the intermediate stimulus (S2). Proportion correct decreased with S1 and increased with S2, but a two-way repeated measures ANOVA showed only a significant effect of S1, $F(2, 8) = 6.9$, $p = .02$.

The accuracy of a frequency discrimination is likely to depend on the differences and the absolute values of the frequencies involved. A difference of three should be easy to detect when the frequencies are three and zero but is hard when they are 103 and 100, for example. The bottom panels confirm this expectation. The left panel shows that proportion correct increased linearly with the difference between S2 and S1 (each point is the average of all pairs for which $S2 - S1$ is equal). The intercept of the regression line, close to .5, suggests that the birds chose mainly between S1 and S2, particularly when the frequencies of these two stimuli were similar; the high frequency of S3 may have essentially excluded it from the choice set. An analysis of the birds’ error patterns supports this in-
Fig. 4. Top: proportion of recency-type errors during the three phases of Experiment 2. Recency-type errors occurred when the correct stimulus was presented in Positions 7 or 8 and the birds avoided the last stimulus. Bottom: proportion of primacy-type errors during Phases 1 and 2. These errors occurred when the correct stimulus was presented in the last position and the birds avoided the first stimulus. (During Phase 3, the total number of errors was so small that no reliable estimate of primacy-type errors was possible for that phase.) The rightmost bars show averages across birds. The details of each phase are the same as in Figure 3.
interpretation because 70% of the total errors consisted in choosing the intermediate stimulus, S2. Stated differently, the birds chose the S1, S2, and S3 stimuli in 77.8, 15.6, and 6.7% of the generalization trials, respectively.

The bottom right panel shows the effect of the absolute frequency of S2 with the difference (S2 - S1) as a parameter. In all three cases in which more than one data point was available, error proportion increased with the absolute frequency of S2. We conclude that the birds’ performance generalized to new stimulus frequencies and that, as expected, accuracy, although always above chance levels, decreased as the stimulus frequencies became more similar.

The present experiment was designed to find out whether pigeons could learn to weigh more equally stimuli that occur at different serial positions, a necessary condition for pure frequency discrimination. The study also addressed the question of whether accurate performance would generalize to new and larger sets of stimulus frequencies. The overall proportions of correct responses, always above chance levels, as well as the generalization data allow a clear affirmative answer to the last question. As for the first question, however, the serial position curves and the pattern of errors suggest that only when the last elements of the sample were constrained was the birds’ behavior independent of the serial position of the stimuli. During the remaining phases without the constraint the results were less clear: If some suggest that the birds were weighing equally
stimuli that occurred at different times (cf. relatively flat serial position curve during Phase 3), the pattern of errors shown in Figure 4 did not always seem to support this conclusion.

EXPERIMENT 3

Experiment 2 did not allow a direct assessment of the effect of the constraint on the control exerted by the first elements of the sample because the least frequent stimulus was presented only in Positions 7 to 9. Experiment 3 attempted to correct this shortcoming by presenting the correct stimulus twice, once in the constraint region and once in the first positions of the sample.

The logic of the experiment remained similar to Experiment 2 except that the sample contained 12 stimuli, with partial frequencies of two, five, and five. During the first phase of the experiment, the last three stimuli were one of each kind, which means that the correct stimulus occurred once in Serial Positions 1 to 9 and once in Serial Positions 10 to 12. During the second phase, the constraint was eliminated. If our interpretation of the effects observed in Experiment 2 is correct, we should not observe a negative recency effect during Phase 1 because the constraint decreases the control of the choice response by the last stimuli of the sample; in Phase 2, however, the recency effect should dominate because without the constraint the last stimuli can control the choice response.

Furthermore, by extending the study of frequency discrimination to larger sets of stimuli, Experiment 3 allowed us to study how negative primacy and recency effects interact. For example, does error rate increase when the correct stimulus is both the first and the last of the series? Are the negative recency and primacy effects enhanced when the last or first two elements of the sample are the correct stimulus? These issues are critical to our understanding of the discriminative function of stimulus frequency and its relation to temporal variables.

Finally, during the third phase of the experiment we included among the regular trials generalization probes that were 20 stimuli long. If the birds had learned a frequency discrimination during Phases 1 and 2, then accurate choice performance should generalize to these new and larger samples.

METHOD

Subjects and Apparatus

Four of the 5 birds of Experiment 2 participated in this study (Bird 219 became sick and was dropped from the experiment). The operant chamber was the same as in Experiment 2.

Procedure

Experiment 3 began immediately after Experiment 2 and followed the same sequence of events. In Phase 1 the sampling period had frequencies of two, five, and five, with the constraint that the last three stimuli had to be one of each kind, in random order. A possible sample is

```
GRRBRRBBBBGRR
```

No signal separated the first nine stimuli from the last three. This phase lasted for 25 sessions. Next, we eliminated the constraint and allowed the two instances of the correct stimulus to appear randomly in any position. This second phase lasted for 26 sessions. Finally, in Phase 3 we interspersed 12 generalization trials among the regular trials of each session. The frequency sets used on these generalization trials, always 20 stimuli long, are presented in Table 2. All other procedural details remained as in Experiment 2.

RESULTS AND DISCUSSION

The proportion of correct choices remained approximately constant throughout the experiment. Averaged over the last five sessions, proportion correct ranged from .67 to .80 during Phase 1, .68 to .84 during Phase 2, and .67 to .82 during Phase 3 (excluding generalization trials).

Serial position curves. In Phase 1 the correct stimulus occurred twice, once in Positions 1 to 9 and once in Positions 10 to 12. A two-way repeated measures ANOVA based on the last 13 sessions of Phase 1 yielded a significant effect of the position of the first occurrence of the stimulus, $F(8, 24) = 2.68$, $p = .03$, but no effect of the position of its second occur-
Fig. 6. Top: Average proportions of correct choices as a function of the serial position of the first (1 to 9) and second (10 to 12) instances of the correct stimulus. The data are from Phase 1 of Experiment 3 (with constraint). Each symbol corresponds to a different bird, and the solid line shows the average across birds. Bottom: similar data from Phase 2 (without constraint).

Fig. 6 illustrates these findings. The symbols to the left of the dotted line show proportion correct for each bird as a function of the first position of the correct stimulus; the symbols to the right show the proportions as a function of the second occurrence of the correct stimulus. To illustrate, a data point at Position 2 corresponds to the average of three proportions: When the correct stimulus appeared in Positions 2 and 10, 2 and 11, and 2 and 12. Similarly, a data point at Position 11 is the average of nine proportions, 1–11, 2–11, . . . , 9–11. Each single proportion was determined from an average of 20 trials. The solid lines show the average across birds. The relative flatness of the two solid lines, except

\[
F(2, 6) = 0.03, \text{ or the interaction between the two variables, } F(16, 48) = 0.89.
\]
at the first two serial positions, indicates a negative primacy effect but no negative recency effects.

During the second phase, the two unconstrained occurrences of the least frequent stimulus yielded a total of 66 pairs of positions. However, in contrast with Phase 1, the first and second positions of the stimulus were no longer independent. For this reason we restricted the analysis of the data to the subset of pairs that occurred in Phase 1. The bottom panel of Figure 6 displays the resulting serial position curves. Each single proportion was computed from an average of 17 trials and used the data from all 26 sessions. Although not completely eliminated, the negative primacy effect observed in Phase 1 was attenuated during Phase 2 (cf. the solid lines for Serial Positions 1 to 9 in the two panels). Moreover, a negative recency effect is now clearly visible (see solid line for Serial Positions 10 to 12 in the bottom panel). A two-way repeated measures ANOVA confirmed the preceding interpretation. In contrast with Phase 1, only the second occurrence of the correct stimulus had a reliable effect on proportion correct, $F(2, 6) = 6.64$, $p = .03$.

The effects of the two positions of the correct stimulus during Phase 2 may be further understood by looking at Figure 7. The filled circles show proportion correct when the first instance of the least frequent stimulus oc-
curved in Position 1, whereas the position of the second instance varied from 2 to 12. We label this curve 1–X (the second occurrence is the variable). The open circles show the opposite case, labeled X–12, in which the position of the second occurrence was fixed at 12 and the position of the first occurrence varied from 1 to 11. Note that the leftmost open circle equals the rightmost filled circle because they both correspond to Pair 1–12. The horizontal lines show the average proportion of a correct choice.

If we look at the X–12 curves we see that, in general, the four curves lie below the horizontal line. This means that when the correct stimulus was presented last, proportion correct was below the bird’s average, another manifestation of the negative recency effect. However, for most of their range these curves do not show any obvious trend. That is, when the correct stimulus was the most recent, the position of its first occurrence had no systematic effect. A one-way repeated measures ANOVA yielded a nonsignificant effect of the first stimulus position, $F(10, 30) = 0.82$. We conclude that no negative primacy effect was visible during Phase 2.

If we now look at the 1–X curves, we see that for all birds the curve decreases with stimulus position. That is, proportion correct decreased when the second instance of the correct stimulus approached the choice period. A one-way repeated measures ANOVA confirmed this negative recency effect, $F(10, 30) = 2.48$, $p = .03$.

Three pairs of stimulus positions deserve special attention. When the correct stimulus occurred in Positions 11 and 12 its recency was greater than in any other case. If pigeons chose on the basis of recency alone and if recency is enhanced by repeating a stimulus, then proportion correct should have been lowest on those trials. The rightmost open circles show that this did not occur. Although such data points are below the overall average for 3 pigeons (excluding 10490), they are not systematically lower than the remaining open circles. A second important pair of positions occurred when the correct stimulus was the first and the second in the sample (its recency was minimal but its primacy was maximal). The leftmost filled circles indicate that in this instance proportion correct varied substantially across pigeons, from above (Bird 2738) to below (Bird 8483) the bird’s overall average. A third important pair of stimulus positions occurred when the correct stimulus was both the first and the last in the sample. If primacy and recency effects combine additively, then proportion correct should have decreased in this case. As the leftmost open circles (or the rightmost filled circles) show, only Bird 10490 clearly confirmed this prediction.

In summary, without the constraint (a) there was no consistent effect of the first occurrence of the correct stimulus; (b) there was no evidence of an additive effect of the two occurrences of that stimulus; and (c) the negative recency effect was the dominant feature of the data.

**Error patterns.** Figure 8 shows the proportions of recency-type and primacy-type errors during the two phases of the experiment (Appendix C provides the raw data). As for the serial position curves, the analysis of the data from Phase 2 was restricted to the samples in which the first and second occurrences of the correct stimulus happened in Positions 1 to 9 and 10 to 12, respectively.

Figure 8 shows that with the constraint in effect recency-type errors were close to chance whereas primacy-type errors were slightly above chance. Removing the constraint increased the recency-type errors, but it did not change the primacy-type errors consistently. A $t$ test for paired samples showed a significant difference between the two phases for recency-type errors, $t(3) = -14.1$, $p = .001$, but not for primacy-type errors $t(3) = .73$.

**Generalization data.** Figure 9 displays the generalization data from Phase 3. The top left panel shows that all pigeons performed above the one third chance level. Averaged across sessions and birds, percentage correct was 78.5%. The top right panel shows that proportion correct decreased with the frequency of the least frequent stimulus (S1) but the intermediate stimulus (S2) had no consistent effect. A two-way repeated measures ANOVA showed a significant effect of S1, $F(2, 6) = 13.78$, $p = .007$, but no effect of S2, $F(2, 6) = 0.09$, or their interaction, $F(4, 12) = 1.19$, $p = .37$.

Proportion correct increased with the difference between the frequencies of S1 and S2 (see bottom left panel). The parameters of
the regression line are similar to those obtained in Experiment 2. In particular, the intercept, close to 0.5, suggests again that the pigeons were choosing only between S1 and S2 when the frequencies of these two stimuli were similar. In fact, 81.2% of the total errors consisted in choosing the intermediate stimulus (S1, S2, and S3 were chosen on 78.5, 16.7, and 4.9% of the generalization trials, respectively). For a constant difference, error proportion also increased with S2 (bottom right panel).

Experiment 3 was designed to assess directly the effects of the constraint on the controlling function of the first stimuli of the sample. In addition, it provided a systematic replication of Experiment 2 with regard to the role of the constraint on the negative recency effect and the generalization of frequency discrimination to larger stimulus sets. Concerning the first goal, the results were somewhat ambiguous because (a) if the average serial position curves and their statistical analyses suggested a slight negative primacy effect when the constraint was operative and its attenuation when the constraint was removed (see Figures 6 and 7), these findings were not corroborated by corresponding changes in the proportion of primacy-type errors (Figure 8, bottom panel); (b) the negative primacy effect, even if real, was certainly small, and therefore much larger data sets will be required in future experiments to detect it reliably.

On the other hand, the results concerning the recency effect reproduced the major findings of Experiment 2, namely, the absence of the negative recency effect with the constraint and its development when the constraint was eliminated. Moreover, the overall proportion of correct choices and the generalization data show that pigeons can discriminate the relative frequencies of three events in large samples of 12 stimuli and that such discrimination generalizes to even larger samples of 20 stimuli.

EXPERIMENT 4

The preceding experiments contained procedural features whose effects on frequency discrimination remain unclear. Two of these features are the long ITI and the fixed relationship between the color and the spatial location of the stimuli. The next experiments ask how frequency discrimination is affected when color is the only relevant stimulus dimension (Experiment 4) and the ITI is eliminated (Experiment 5).

In Experiments 1 to 3 the red light was always presented on the left key, the green light on the right, and the blue or white light on the center. Furthermore, the birds had to raise or lower their heads to peck the center key, and move right and left to peck the side keys. Therefore, the color of the keys, their spatial location, and even the pigeons' differential body movements could have controlled performance during the choice phase. Experiment 4 attempted to isolate the effects of stimulus color on frequency discrimination.
For that purpose, all sample stimuli were presented on the center key. During the choice period, the bird chose between two side keys, one illuminated with the least frequent stimulus (the correct choice), and the other with one of the remaining (incorrect) stimuli. If pigeons can learn a frequency discrimination on the basis of color alone, then eliminating positional cues and differential body movements should not diminish their performance.

**METHOD**

**Subjects**

Four experimentally naive pigeons (*Columba livia*) participated in the experiment. Housing conditions were similar to those in Experiments 2 and 3.

**Apparatus**

A standard Med Associates operant chamber for pigeons was used. The chamber was 29 by 24 cm along the sides and 30 cm high. The front aluminum wall contained three response keys in a linear arrangement, each 2.5 cm in diameter and 8 cm apart, center to center. The keys were located 24 cm above the floor, and the side keys were 2.5 cm from the side walls. All other details of the experimental chamber and outer box were as in Experiment 3.

**Procedure**

**Pretraining.** The autoshaping and transition phases were similar to those of the preceding experiments except for the following changes: During each autoshaping trial, one
of the three keys was randomly selected and then illuminated with a randomly selected color (red, green, or blue). After five sessions all birds were pecking the keys reliably, regardless of their color. During the transition phase, each session started with the illumination of the left key with a randomly selected color. A peck at the illuminated key provided either a 3-s reinforcer or a 0.4-s delay during which all keys were turned off. Pecks during the delay reset the timer for the interval. After the delay or the reinforcement episode, the same key was illuminated with a randomly selected light. After receiving 14 reinforcers from the left key, the bird was shifted to the center key and the procedure was repeated. When 14 reinforcers were obtained from the center key, the bird was shifted to the right key. Sessions ended when the bird had earned 42 reinforcers. Across five sessions, the probability of reinforcement per peck decreased from .5 to .08.

The experiment was divided into three phases. In Phase 1, which lasted from 26 to 29 sessions, the frequencies of the three stimuli were zero, three, and three. In Phase 2, which lasted for 30 sessions, the frequencies increased to one, four, and four and the least frequent stimulus could appear in any one of the nine positions (no constraint was in effect). During Phase 3, which lasted for 10 sessions, 12 generalization trials were interspersed among the 1–4–4 regular trials. However, in contrast with Experiments 2 and 3, correct choices during the generalization tests were not reinforced. The sets of stimulus frequencies used during the generalization tests were the same as those used in Experiment 2 (see Table 2).

**Results and Discussion**

Averaged over the last five sessions, the proportion of correct responses ranged across birds from .79 to .91 in Phase 1, .84 to .90 in Phase 2, and .81 to .90 in Phase 3 (excluding generalization trials). All these values are above the 99% upper confidence limit associated with random responding.

To determine the influence of recency on frequency discrimination during the 3–3–0 phase, we divided the birds’ errors into two groups according to whether the stimulus presented with the correct stimulus during the choice period was the last stimulus of the sample (L) or the remaining stimulus (O). A recency effect would predict a greater error rate in the latter case. The proportions were .083(L) and .084(O) for Bird 10417, .19(L) and .21(O) for Bird 10413, .14(L) and .17(O) for Bird 8554, and .13(L) and .25(O) for Bird 5155. Although in the predicted direction, the effect was small for 3 of the 4 pigeons.

**Serial position curves.** Figure 10 shows the average proportion of correct choices in the last eight sessions of Phases 2 and 3 as a function of the serial position of the least frequent stimulus (Appendix D gives individual data). The two phases yielded similar serial position curves. Proportion correct decreased slightly as the least frequent stimulus was closer to the choice period—a negative recency effect. There is also evidence of a slight primacy effect in both curves. For both phases, a one-way repeated measures ANOVA showed a significant effect of the serial position of the least frequent stimulus, Phase 2: $F(8, 24) =$
Generalization data. Figure 11 displays the generalization results from Phase 3. The top left panel shows that, with few exceptions, performance was above the 50% chance level and showed no trend across sessions. Averaged across sessions and birds, percentage correct was 84.4%. The similarity of the curves in Figures 5, 9, and 11 suggests that the decision to reinforce (Experiments 2 and 3) or not to reinforce (Experiment 4) correct responses during the tests had no effect on choice accuracy.

The top right panel shows that when the
intermediate stimulus (S2) occurred four times, proportion correct decreased with the frequency of the least frequent stimulus (S1). However, the other two curves suggest a weak interaction between the frequencies of S1 and S2. In fact, a two-way repeated measures ANOVA yielded significant effects of S1, $F(2, 6) = 5.23, p = .049$, and its interaction with S2, $F(4, 12) = 4.01, p = .027$, but no main effect of S2, $F(2, 6) = 2.5, p = .16$.

As in previous experiments, proportion correct increased linearly with the difference, $S_2 - S_1$. The value of the intercept, close to .25, may be explained as follows. If, as we argued before, the birds have little difficulty in discriminating the frequency of S3, then on trials in which S1 is presented with S3, the probability of a correct choice is likely to be close to 1. On the remaining trials S1 is presented with S2, and when their frequencies are equal, the proportion of correct choice is likely to be .5. Therefore, averaged over the two types of trials, proportion correct should be close to .75. In fact, when S1 was paired with S3, proportion correct averaged .91 (range, .81 to .96); when S1 was paired with S2, proportion correct averaged .77 (range, .74 to .82). The low number of total errors did not allow their further partition into different categories.

Finally, the bottom right panel of Figure 11 shows the effects of the absolute frequency of S2 when the difference $S_2 - S_1$ is held constant. When $S_2 - S_1 = 2$ or 4, proportion correct decreased with S2, as in previous experiments. But, when $S_2 - S_1 = 3$, proportion correct increased with S2. The different trends of the three curves are another manifestation of the interaction effect revealed by the ANOVA. Although we have no adequate explanation for the preceding interaction, we conclude that when color was the only stimulus dimension the birds’ performance generalized to new sets of frequencies.

**EXPERIMENT 5**

Experiments 1 to 4 included a 20-s ITI to reduce the potential proactive interference from one trial to the next. However, the serial position curves (e.g., Figure 2) suggest that stimulus control decays so rapidly with time that proactive interference may not play an important role in frequency discrimination. Experiment 5 was designed to find out whether accurate performance could be maintained with shorter ITIs. In addition, the first phase of the experiment (a replication of Phase 1 of Experiment 1) provided additional information on how pigeons learn a frequency discrimination in which one stimulus has a zero frequency.

**METHOD**

**Subjects and Apparatus**

Two naive pigeons (*Columbia livia*) participated in the experiment. The experimental chamber and the housing conditions were the same as in Experiment 1.

**Procedure**

The birds were initially exposed to the preliminary training described in Experiment 1. During the frequency discrimination training, the frequencies of the three stimuli in the sampling period were always zero, three, and three. The only variable that changed across the three phases of the experiment was the duration of the ITI. During Phase 1, the ITI remained constant at 20 s. During Phase 2, the ITI was titrated as follows: Starting with a 10-s duration, each correct response decreased the ITI by 1 s whereas each incorrect response increased it by 4 s, up to a maximum of 20 s. In Phase 3, the ITI was eliminated; a new trial began immediately after reinforcement if the choice was correct, and immediately after a 4-s timeout if the choice was incorrect. All other procedural details remained the same as in Experiment 1.

**RESULTS AND DISCUSSION**

The overall proportion of correct choices was above chance levels after the first session, and reached asymptotic values comparable to those obtained in Experiment 1. When averaged over the last five sessions of each phase, the proportions of correct choices were .90, .88, and .89 for Bird 2097 and .83, .84, and .87 for Bird 7323. Titrating the ITI (Phase 2) or removing it (Phase 3) did not affect performance. We conclude that once the 3–3–0 discrimination is learned, the ITI is not necessary to maintain it. It remains to be seen whether the ITI plays a significant role during the acquisition of the discrimination or the discrimination of larger frequency sets.
GENERAL DISCUSSION

Our five experiments show that (a) pigeons can learn to discriminate the relative frequency of three stimuli; (b) when the stimuli are presented in random order, the likelihood of a correct choice decreases with the serial position of the correct stimulus, a negative recency effect; (c) when the last three elements of the sample are constrained to be one of each kind, the negative recency effect decreases but errors may become more likely when the correct stimulus occurs early in the sample, a negative primacy effect; (d) under some conditions the probability of a correct choice is independent of the serial position of the correct stimulus; and (e) accurate performance generalizes to new and larger samples, up to $N = 20$. In what follows, we discuss our results in relation to three issues: how relative frequency becomes a dimension of stimulus control, how pigeons integrate events over time, and how frequency discrimination may underlie other types of performance.

Frequency As a Dimension of Stimulus Control

In general, accurate frequency discrimination developed quickly, particularly when the stimuli differed in color and spatial location (e.g., Figure 1). For some birds performance was significantly above chance levels even during the first session. Such rapid learning may be due to the effects of extinction: If each nonreinforced peck during the sampling period decreases the probability of pecking the same key, then the likelihood of pecking the least frequent key during the choice period will be greater than the probability of pecking the other keys. After extended training, however, extinction is likely to have negligible effects, and therefore it is less likely to account for asymptotic performance.

The high proportion of correct choices clearly shows that pigeons can discriminate the relative frequency of events, but that measure gives no clues about the process or processes that underlie such performance. For example, how does the serial order of the stimuli influence the discrimination of their relative frequencies? How do constraints on training sequences change the controlling functions of serially presented stimuli? What does performance during generalization probes tell us about the relative influence on choice of the different stimuli? To answer these and related questions we need to analyze how choice is affected by the position of the stimuli (serial position curves plus error patterns) and by their specific frequencies (generalization data). We address the serial position curves and the error patterns first and then discuss the generalization data.

Serial position curves: Negative recency. When stimuli are presented in random order but with different frequencies, it is necessarily the case that on average the least frequent stimulus is also the least recent one. In other words, random presentations entail a negative correlation between the last elements of the sample and the correct choice. Under such experimental conditions, and in agreement with Alsop and Honig's (1991) findings, we observed in Experiments 1 and 4 a decreasing proportion of correct choices as the least frequent stimulus came closer to the choice period. This negative recency effect suggests that pigeons learn to avoid recent stimuli to solve frequency discrimination tasks. Hence, in Experiments 2 and 3 we attempted to reduce the negative recency effect by constraining the last three elements of the sample to be one of each kind. We hypothesized that the constraint would prevent the last stimuli from gaining exclusive (avoidance) control over the choice response, and, as a consequence, control by the other stimuli would increase. The more general reason to study the effects of the constraint was that a pure frequency discrimination requires equal weighing of all events.

As predicted, Experiment 2 (see Figure 3) showed no evidence of unequal weighing of the sample stimuli when the constraint was operative, and showed a clear negative recency effect when the constraint was removed. Surprisingly to us, however, it also showed that when generalization tests were included in the session, the negative recency effect disappeared. Similarly, when the constraint was operative in Experiment 3 (see Figure 6) no negative recency effect was observed; when the constraint was eliminated, the negative recency effect was clearly visible. Even if not perfect, the observed variation of the recency effect with the presence or absence of the constraint suggests that recency effects may
be due not only to the structure of memory (forgetting implies greater weight to more recent events) but also to the functional properties of the task such as the correlation between the last elements of the sample and the choice response.

*Serial position curves: Negative primacy.* It is reasonable to assume that when the constraint broke the correlation between the last three stimuli and the correct choice response, the birds paid more attention to the beginning of the sample. First, by avoiding the first stimuli they would insure a high probability of reinforcement, as was observed. Second, avoidance of the initial stimuli would also explain the strong, but transient, negative primacy effect observed when the constraint was removed in Phase 2 of Experiment 2 (see the open circles in the middle panel of Figure 3). Third, if the constraint is indeed responsible for the negative primacy effect, then we would have expected the strengthening of this effect when the constraint was reintroduced in Phase 1 of Experiment 3 (see Figure 6), as well as its weakening when the constraint was subsequently eliminated. As we mentioned before, to some extent these predictions were also observed. Reasonable as it may be, however, the preceding interpretation is not corroborated by the birds' error patterns, because the proportion of primacy-type errors did not covary with the slopes of the corresponding serial position curves. In Figure 4, for example, primacy-type errors were never consistently above chance even when the serial position curves indicated a strong negative primacy effect; in Figure 8 primacy-type errors did not change between conditions even though the serial position curves show that the negative primacy effect decreased slightly when the constraint was removed.

The occasional independence of primacy and recency effects from recency-type and primacy-type errors raises some conceptual issues. For example, should primacy and recency effects be defined by the slopes of the serial position curves, that is, by the changes in proportion correct as a function of the position of the correct stimulus, or by the types of errors produced when the other stimuli occur in the last or first positions? More important, perhaps, what are the consequences of this occasional independence to our definitions of frequency discrimination?

Initially we proposed that a pure, uncontaminated relative frequency discrimination occurs when performance during the choice period (a) is clearly above chance levels, otherwise we cannot speak of discrimination; (b) generalizes to new samples, otherwise we cannot rule out the hypothesis that performance is due to specific stimulus–response pairings; and (c) indicates that all stimuli are given equal weights, otherwise the frequency of a stimulus will be underestimated depending on its serial position. Determining whether a bird has met the first two conditions is a straightforward matter, but our initial proposal that the final condition is met when the serial position curves are flat is, in light of our results, insufficient; the proportion of recency- and primacy-type errors must also remain at chance level. When any of these two tests are failed, the birds may be discriminating the frequency of the stimuli, but their discrimination is also being influenced by other variables. The birds in the present study always satisfied the first two conditions (the least stringent), but the final condition, properly assessed, was satisfied only in Phase 1 of Experiment 2. In all other phases either the serial curves were not flat or the primacy- or recency-type errors were not at chance levels. In summary, after taking into account our entire set of results, we predict that pure frequency discrimination, as we defined it above, will be obtained only when constraints at both ends of the list of the sample eliminate differential weighing of the stimuli according to their serial position. This prediction remains to be tested.

Another question raised by our findings concerns the type of behavioral process that might explain the observed serial position effects. For example, could the negative primacy and recency effects be due to the poorly understood phenomena of proactive and retroactive interference (Underwood & Postman, 1973; Wright, Urcioli, & Sands, 1986)? If each stimulus disrupts to some extent the controlling function of antecedent stimuli (retroactive interference) and subsequent stimuli (proactive interference), then the serial position curves should have an inverted U shape. The prediction follows because the middle stimuli suffer from both types of in-
The results from the three sets of generalization trials may be described by the following equation

$$
PC = b_0 + b_1S1 + b_2S2,
$$

where $PC$ is the probability of a correct choice, $S1$ and $S2$ are the frequencies of the least frequent and the intermediate stimuli, respectively, and $b_0$, $b_1$, and $b_2$ are three parameters. The least squares estimation of these parameters for our generalization data is shown in Table 3. The $\beta$ values are the corresponding parameters when the three variables are standardized to have zero mean and unit variance.

The negative values of $b_1$ indicate that, for fixed $S2$, proportion correct decreased with $S1$. The positive values for $b_2$ in Experiments 3 and 5 indicate that, for fixed $S1$, proportion correct tended to increase with $S2$ ($b_2$ was essentially 0 in Experiment 3). The ratio $\beta_1/\beta_2$ is a measure of the relative influences of $S1$ and $S2$ on choice behavior (e.g., Kachigan, 1991), shows that $S1$ contributed substantially more to choice performance than $S2$.

As explained in Appendix E, Equation 1 also predicts that

$$
PC = \frac{b_2 - b_1}{2}(S2 - S1) + c,
$$

where $c$ is a constant. That is, the probability of a correct choice should increase linearly with the difference $S2 - S1$ (provided that $b_2 > b_1$, which was always the case). All generalization tests yielded data in agreement with Equation 2. In fact, the slopes of the regression lines shown in the bottom left panels of Figures 5, 9, and 11 agree with the predictions of Equation 2 (.08 vs. .08 for Experiment 2, .06 vs. .07 for Experiment 3, and .05 vs. .05 for Experiment 4).

Equation 3 also predicts a direct relation between proportion correct and $S2$ when the difference, $S2 - S1$, is held constant:

$$
PC = (b_1 + b_2)S2 + b_0 - b_1k,
$$

where $k = S2 - S1$. When $S1$ is negative and greater than $S2$ in absolute value, Equation 3 predicts a decrease in $PC$ with $S2$. The three sets of generalization data only partially supported Equation 3, because if proportion correct generally decreased with $S2$ (for constant $k$), the lines did not always have the same slope (see bottom right panels in Figures 5, 9, and 11).

In summary, as the $r^2$ values in Table 3 suggest, Equation 1 was strongly supported in Experiment 2, somewhat supported in Experiment 3, and less supported in Experiment 4. It remains to be seen whether the observed deviations from Equation 1 will hold in future studies in which (a) more trials are used to estimate the probabilities involved here and (b) the size of the stimulus sample is increased.

### Table 3

<table>
<thead>
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<th>Experiment 3</th>
<th>Experiment 4</th>
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Memory and the Temporal Course of Stimulus Control

The shape of our serial position curves shed some light on how pigeons weight events with different ages. A full understanding of this weighing process is a necessary condition to predict the circumstances under which pigeons perform accurate frequency discriminations. In what follows we restrict our analyses to the 4–3–1 (Experiment 1) and 4–4–1 (Experiments 2 and 4) conditions, because in the remaining conditions the correct stimulus either did not occur, in which case the data are not directly relevant to the weighing issue, or occurred more than once, in which case the weighing function is certainly more complex and difficult to interpret.

One theoretical interpretation of the negative recency effect would claim that the birds learned to choose the stimulus with the weakest memory trace at the beginning of the choice period (e.g., Wickelgren & Norman, 1966). If the correct stimulus occurs early in the sample, then by the time the choice is made its memory trace will be much weaker than if the stimulus occurs late in the sample. The simplest quantitative formulation encompassing this idea is the exponential decay model (e.g., Killeen, 1994; Wixted, 1990),

$$PC(t) = 1 - Be^{-\gamma(T-t)} \quad (0 \leq t \leq T), \quad (4)$$

where PC(t) is the probability of choosing the correct stimulus when it occurred at time t, B and \( \gamma \) are constants, and the term \( Be^{-\gamma(T-t)} \) represents the strength of the memory trace at the moment of choice, T. Because the interstimulus interval was approximately constant in our experiments, we assume that stimuli occurred at times \( t = 1, 2, \ldots, T \) with \( T = N \), the sample size.

A few serial position curves from the 4–3–1 or 4–4–1 conditions support the exponential model. In the top panels of Figure 12 we plot two examples and the corresponding best fitting exponential functions. However, other serial position curves from the same conditions clearly disconfirm the simple decay model because, in addition to the negative recency effect, they also show a negative primacy effect. A simple exponential never predicts a negative primacy effect.

One way to account for both negative primacy and recency effects is to assume that two different processes determine the trace strength of a stimulus. One, the familiar decay process, underlies the common observation that stimuli lose their effectiveness with the passage of time; this process can be described by Equation 4. The other, what we tentatively call a salience or distinctiveness process, describes how different variables may affect the residual strength of a stimulus. For example, constraints on the last stimuli may shift the bird’s attention to the beginning of the sample and, as a consequence, the first stimuli may become relatively more effective at controlling the choice response. Similar effects may also be produced by decreasing the duration of successive sample stimuli (Shimp, 1976), by proactive interference (Wright et al., 1986), or by changing the similarity between the context of successive stimuli and the context of choice (Donahoe & Palmer, 1994). Together or in isolation, these factors may cause a decrease in the residual salience of a stimulus with its ordinal position, the negative primacy effect.

The following equation can describe how the salience and decay processes codetermine the controlling function of a stimulus, PC(t):

$$PC(t) = 1 - [Ae^{-\lambda t} + Be^{-\gamma(T-t)}], \quad (5)$$

where \( \lambda \) and \( \gamma \) are decay constants, and A and B are weight constants. The memory trace, the term in brackets, is influenced by two processes, the familiar passive decay process described by \( Be^{-\gamma(T-t)} \) and the new residual salience process described by \( Ae^{-\lambda t} \). Equation 5 has the following properties. First, when \( A = 0 \) it reduces to Equation 4. Second, when \( A > 0 \) the salience of the correct stimulus will remain greater than 0 regardless of how many Gs and Bs are subsequently presented; hence the term residual salience. Finally, when A and B are small and \( \gamma \) and \( \lambda \) are relatively large, Equation 5 approximates a horizontal line, that is, a flat serial position curve.

The six bottom panels in Figure 12 replot the data of 6 birds and the best fitting curves based on Equation 5. The curves for the remaining pigeons in the 4–4–1 condition were too noisy to justify a quantitative fit. The the-
Fig. 12. Fit of Equation 5 to individual serial position curves. In the two top panels $\lambda = 0$. In the four middle panels $\gamma = \lambda$. In the two bottom panels $\gamma \neq \lambda$. All data sets come from experimental conditions in which the correct stimulus occurred only once. Open and filled circles come from Phases 2 and 3 of Experiment 2 (Birds 9882, 10490, and 219) and Experiment 4 (Birds 10413 and 10417), respectively.
Fig. 13. Left: Based on Equation 5, the curves show the strength of the memory trace as a function of time when the stimulus occurs in Positions 0 (solid) or 5 (dotted). Right: Strength of the memory trace at the moment of choice for stimuli that occur in Positions 1, 2, 3, and 4, relative to the memory trace of the stimulus in Position 1. Each set of four connected data points corresponds to a different retention interval. As this interval increases, the recency effect is replaced by the primacy effect.

Theoretical curves reproduced the major trends in the data. In the four middle panels the fit remained good when we assumed that $\gamma = \lambda$. The data from Birds 10417 and 10490, on the other hand, required $\gamma \neq \lambda$.

Although the number of parameters increased from two in Equation 4 to four in Equation 5, we believe that Equation 5 provides a convenient descriptive tool to analyze the influence of temporal variables in frequency discrimination tasks. It also makes interesting predictions. For example, if we increase the time between the end of the sampling period and the beginning of the choice period (the retention interval), the effect of stimulus salience increases in comparison with the effect of passive decay (assuming $A > 0$). This happens because, for fixed $T$, the passive decay term goes to 0 but the stimulus salience term remains constant. Hence, Equation 5 predicts that for large $T$ the primacy effect should increase relative to the recency effect.

Figure 13 illustrates the prediction. For convenience, we have used Equation 5 with $\lambda = \gamma$. The left panel shows the strength of the memory trace when the correct stimulus occurs in Position 0 (solid curve) or five positions later (dotted curve). The critical feature to note is that the curves intersect. For values of $T$ less than 10 the trace of the stimulus when it occurs in Position 0 is less than when it occurs in Position 5; however, for large $T$ the converse is true. If the animal selects the stimulus that at the moment of choice has the weakest memory trace, then as the retention interval increases, the likelihood of an error should increase with the age of the stimulus—the negative recency effect should give place to the primacy effect. Such a reversal is illustrated in the right panel. The graph shows the traces when the correct stimulus occurs in Positions 1, 2, 3, or 4, relative to the memory trace when the stimulus occurs in Position 1. Each set of four connected points corresponds to a different retention interval. We see that as the retention interval increases, the recency effect disappears but the primacy effect increases.

Wright et al. (1985) reported data that are in agreement with the preceding predictions. Pigeons decided whether a comparison slide that they were currently seeing was included
in a sample of four slides that had been presented before. When the slide was included in the sample, the authors observed a positive recency effect for short retention intervals (probability correct increased with the ordinal position of the slide), but only a positive primacy effect at long retention intervals (probability correct was highest when the slide occurred in the first positions).

The foregoing discussion highlights the pervasive influence of temporal variables on frequency discrimination, and it naturally suggests a more radical question: Can frequency discriminations be based exclusively on temporal discriminations? For example, in our least frequent discrimination task, pigeons could have learned to choose the stimulus that at the moment of choice had the shortest cumulative duration. Similarly, in Alsop and Honig’s (1991) task, pigeons could have learned to choose the left key if the total duration of the blue light exceeded that of the red light. Although recent studies (e.g., Roberts, 1995) indicate that the discriminations of time and number are not based on the same mechanism, future studies will need to investigate the issue directly by varying total duration and frequency independently. We will not be surprised if the direction of influence proves to go both ways.

The Potential Generality of Frequency Discrimination

Frequency discrimination may be a major component of more complex behavioral processes. As suggested before, it may play a critical role in concept learning. It may also be critical in the operant conditioning of behavioral variability. For example, Machado (1989, 1992) rewarded pigeons whenever their sequences of four choices differed from the sequences emitted during the last x trials (where x was a schedule parameter). The pigeons generated highly variable, random-like behavior (see also Page & Neuringer, 1985). Although it is still unclear how the birds learned such behavior (see Machado, 1993, for alternative accounts), the present findings suggest one possibility: If the frequency of a response pattern is also a discriminable dimension of behavior (Mechner & Guevrekian, 1962), the birds in Machado’s and similar experiments may have learned that reinforcement was more likely after the least frequent patterns than the most frequent patterns. That is, pigeons may have learned that after many pecks on the right key, left key pecks were more likely to be rewarded; that after many left-right and right-left pairs of key pecks, repeats (left-left and right-right) had higher payoff probabilities, and so on for larger patterns. If the birds’ choices came under the discriminative control of the relative frequency of its response patterns, then their behavior would soon converge to highly variable responding, the observed outcome. As this example further attests, the full generality of frequency discrimination remains to be explored.

REFERENCES


APPENDIX A

Proportion of correct responses in Experiment 2 as a function of the serial position of the correct stimulus. Phase 1 (last seven sessions) = with constraint; Phase 2 (first eight and last seven sessions) = without constraint; Phase 3 (six sessions) = without constraint + generalization tests.

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Final acceptance September 30, 1996
APPENDIX B

Pattern of errors observed in Experiment 2. Number of opportunities to make an error (N), the total number of errors effectively made (E), and the number of recency-type (R) and primacy-type (P) errors. Phase 1: last seven sessions; Phase 2: first eight and last seven sessions; Phase 3: six sessions. S1 is the least frequent stimulus.

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<th>S1 in Position 9</th>
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APPENDIX C

Pattern of errors observed in Experiment 3. Number of opportunities to make an error (N), the total number of errors effectively made (E), and the number of recency-type (R) and primacy-type (P) errors. Phase 1: last 13 sessions; Phase 2: all 26 sessions. S1(1) and S1(2) refer to the positions of the first and second occurrences of the least frequent stimulus, respectively.

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<tr>
<th>Phase</th>
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<th>S1(2) = 12, S1(1) = 2, 3, ..., or 9</th>
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<td>353</td>
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<td>113 (31.6)</td>
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APPENDIX D

Proportion of correct responses in Experiment 4 as a function of the serial position of the correct stimulus. The data come from the last eight sessions of Phases 2 and 3; Phase 3 includes generalization trials.

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APPENDIX E

To derive Equation 2 from Equation 1, we illustrate the reasoning for two specific cases from Experiment 2; the remaining cases are similar and are not shown. The probability of a correct response when the difference between $S_2$ and $S_1$ equals 2 is the average of two probabilities, the probability following the samples in which $S_1 = 1$ and $S_2 = 3$, and the probability following the samples in which $S_1 = 2$ and $S_2 = 4$. No other samples yield a difference of 2 between $S_2$ and $S_1$. Hence, according to Equation 1 the probability of a correct response when $S_2 - S_1 = 2$ is given by

$$P(2) = (b_0 + 1 \times b_1 + 3 \times b_2) + (b_0 + 2 \times b_1 + 4 \times b_2)/2$$

$$= b_0 + 1.5 \times b_1 + 3.5 \times b_2.$$  

In the last expression the difference between the multipliers of $b_2$ and $b_1$ equals 2, and their sum equals 5. For $P(3)$ we use three samples: $S_1 = 0$ and $S_2 = 3$, $S_1 = 1$ and $S_2 = 4$, and $S_1 = 2$ and $S_3 = 5$. Therefore,

$$P(3) = (b_0 + 0 \times b_1 + 3 \times b_2) + (b_0 + 1 \times b_1 + 4 \times b_2) + (b_0 + 2 \times b_1 + 5 \times b_2)/3$$

$$= b_0 + 1 + b_1 + 4 \times b_2.$$  

The difference between the multipliers of $b_2$ and $b_1$ equals 3, but their sum remains constant at 5. More generally, given the range of values for $S_1$ and $S_2$, the probability of an error when $S_2 - S_1 = k$ is

$$P(k) = b_0 + (5/2 - k/2) \times b_1$$

$$+ (5/2 + k/2) \times b_2$$

$$= b_0 + \frac{5(b_1 + b_2)}{2} + \frac{b_2 - b_1}{2}.$$  

This is Equation 2 in the text, with $c = b_0 + 5(b_1 + b_2)/2$.

To obtain Equation 3, rewrite Equation 1 as follows:

$$PC = b_0 + b_1 S_1 + b_2 S_2 + b_1 S_2 - b_2 S_1$$

$$= b_0 - b_1 (S_2 - S_1) + (b_1 + b_2) S_2.$$  

This is Equation 3 in the text with $k = S_2 - S_1$. 

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